Index

Unit- 9.Ray Optics

every day may not be good,

but there's something good in every day.

9. Ray Optics

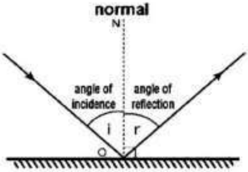
Ray & Ray Optics

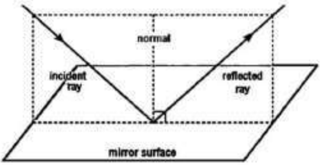
A ray of light is the straight line path followed by light in going from one point to another.

The ray optics therefore uses the geometry of straight lines to represent macroscopic phenomena like rectilinear propagation, reflection, refraction etc.

Reflection of Light

Reflection is the phenomena of change in the path of light without any change in medium.





In figure M_1M_2 is a plane mirror. A ray of light falls on the mirror along AO at $\angle AON = i$ is called incident ray and i is called angle of incidence, where ON is normal to the mirror at O.

Incident ray is reflected along OB at $\angle NOB = r$, It is called reflected ray and r is known as angle of reflection.

Laws of reflection -

(i) Angle of incidence is equal to angle of reflection.

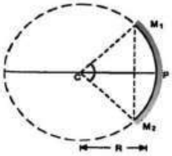
$$\boxed{\angle i = \angle r}$$

(ii) Incident ray (OA), reflected ray (OB) and normal (ON) to the mirror, all lie in the same plane.

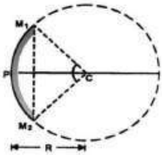
Spherical Mirrors -

A spherical mirror is a part of a hollow sphere, whose one side is reflecting and other side is opaque.

(1) **Concave mirror** - whose reflecting surface is towards the centre of the sphere of which the mirror is a part. As shown in figure (a)



a



b

(2) **Convex mirror** - whose reflecting surface is away from the centre of the sphare of which the mirror is a part. As shown in figure (b).

Some Important Terms

(i) **Centre of curvature (c)** -

The centre of the sphere of which the mirror forms a part is called the centre of curvature. It is represented by 'C'.

(ii) **Radius of curvature (R)** -

The radius of the sphare of which the mirror forms a part is called the radius of curvature of the spherical mirror.
It is represented by 'R'.

(iii) Vertex or Pole-

The middle point or centre of the spherical mirror is called vertex or pole of the mirror. It is represented by P.

(iv) Principal focus (F)-

It is a point on the principal axis of the mirror at which rays incident on the mirror in a direction parallel to the principal axis actually meet or appear to diverge after reflection from the mirror.

(v) Angular aperture -

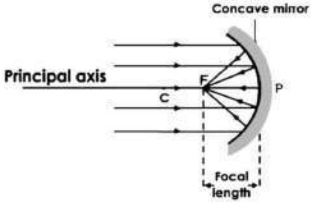
The angle M_1CM_2, subtended at C by the by the diameter of the spherical mirror is called angular aperture.

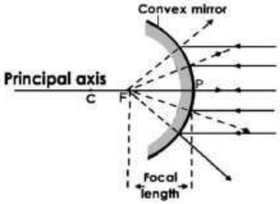
(vi) Principal axis -

The straight line joining the pole and the centre of curvature of spherical mirror extended on both sides is called principal axis of the mirror.

(vii) Principal section-

A section of the spherical mirror cut by a plane passing through pole and centre of curvature of the mirror is called principal section of the mirror.





* The distance of principal focus F from the pole P of the spherical mirror is called focal length (f) of the mirror.

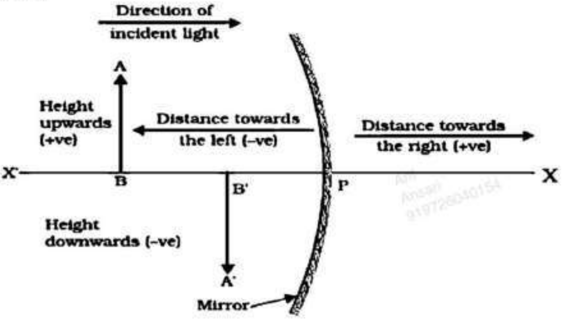
$$\boxed{PF = f}$$

New Cartesian sign Conventions -

(1) All the distances are measured from pole (P) of spherical mirror.

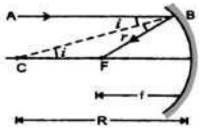
(2) The distances measured in the direction of incidence of light are taken as positive and the distances measured in a direction opposite to the direction of incidence of light are taken as negative.

(3) The heights measured upwards and perpendicular to the principal axis of the mirror are taken positive and the heights measured downwards are taken as negative.

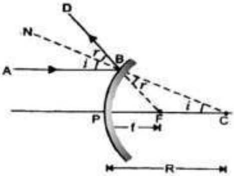


Relation between f and R

Concave Mirror



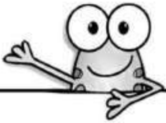
Convex Mirror



* The focal length of a concave or convex mirror is equal to half the radius of curvature of the mirror.

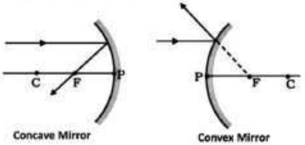
$$\boxed{f = \frac{R}{2}}$$

Some Important Rules



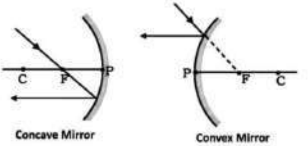
Rule 1 -

Ray parallel to principal axis will pass through focus after reflection



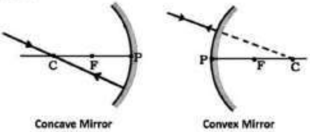
Rule 2 -

Ray passing through focus will become parallel to principal axis after reflection



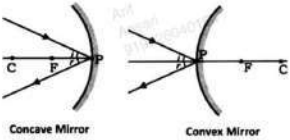
Rule 3 -

Ray which passes through the centre of curvature reflected back along its own path.



Rule 4 -

Ray incident at pole is reflected back making same angle with principal axis

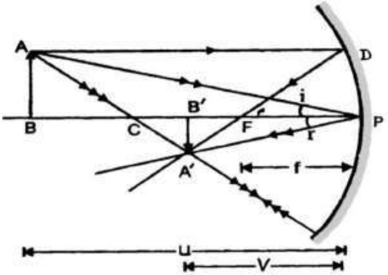


Mirror Formula for Concave Mirror

Mirror formula is a relation between focal Length of the mirror and distances of object and image from the mirror.

(a) Real Image -

Let an object AB is held perpendicular to the principal axis of the mirror beyond C.



A ray of light starting from A and incident on the mirror along AD parallel to the principal axis, gets reflected from the mirror and passes through F.
Another ray of light incident along AP is reflected along PA′ such that $\angle APB = \angle i = \angle BPA' = \angle r$

The two reflected rays actually meet at A′ which is real image of the point A. A third ray starting from A and incident on the mirror along AC falls normally on the mirror and retrace its path, meeting the two reflected rays at A′.

From A′ draw A′B′ perpendicular on the principal axis. Therefore A′B′ is real, inverted image of AB formed by reflection from the concave mirror.

As ΔABC and $\Delta A'B'C$ are similar, so that-

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \quad \text{—} \quad (1)$$

Again as ΔABP and $\Delta A'B'P$ are similar, so that-

$$\frac{AB}{A'B'} = \frac{PB}{PB'} \quad \text{——} \quad (2)$$

From eq. (1) and eq. (2) we get -

$$\frac{CB}{CB'} = \frac{PB}{PB'} \quad \text{——} \quad (3)$$

By measuring all distances from P, we have -

$$CB = PB - PC \quad \text{and} \quad CB' = PC - PB'$$

$\therefore$ from eq. (3) $$\frac{PB - PC}{PC - PB'} = \frac{PB}{PB'} \quad \text{——} \quad (4)$$

Using new cartesian sign Conventions, we get -

$$PB = -u, \; PC = -R \text{ and } PB' = -v$$

so we get from eq. (4), that -

$$\frac{-u + R}{-R + v} = \frac{-u}{-v}$$

or $$uR - uv = uv - vR$$

or $$uR + vR = 2uv$$

by deviding both side by uvR, we get -

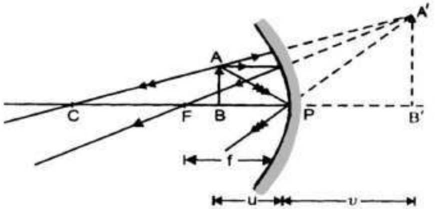
$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$$

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}} \qquad \text{As } f = \frac{R}{2}$$

It is the required mirror formula.

(b) Virtual Image -

When the object is held in front of a concave mirror between the pole P and principal focus F of the mirror, then the image formed is virtual, erect and magnified as shown in figure.



As ΔABC and $\Delta A'B'C$ are similar, so that-

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \quad \text{——} \quad (1)$$

Again as ΔABP and $\Delta A'B'P$ are similar, so that-

$$\frac{AB}{A'B'} = \frac{PB}{PB'} \quad \text{——} \quad (2)$$

From eq. (1) and eq. (2) we get-

$$\frac{CB}{CB'} = \frac{PB}{PB'} \quad \text{——} \quad (3)$$

By measuring all distances from P, we get-

$$CB = PC - PB \quad \text{and} \quad CB' = PC - PB'$$

$$\therefore \quad \frac{PC - PB}{PC + PB'} = \frac{PB}{PB'}$$

Using new cartesian sign conventions-

$$PB = -u \quad , \quad PB' = +V \quad , \quad PC = -R$$

then

$$\frac{-R + u}{-R + V} = \frac{-u}{V}$$

or

$$uR - uV = -VR + uV$$

$$uR + VR = 2uV$$

Dividing both sides by uVR, we get-

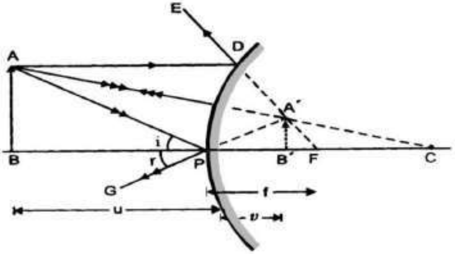
$$\frac{1}{V} + \frac{1}{u} = \frac{2}{R}$$

$$\boxed{\frac{1}{V} + \frac{1}{u} = \frac{1}{f}}$$

It is the required mirror formula.

Mirror Formula for Convex Mirror

The image formed in a convex mirror is always virtual and erect, whatever be the position of the object.



A ray of light starting from A and incident on the mirror along AD in a direction parallel to the principal axis is reflected along DE. On producing back, DE appears to come from F.

Another ray from A incident on the mirror along AP is reflected along PG such that

$$\angle APB = \angle i = \angle BPG = \angle r$$

on producing back, PG meets DF at A'. Therefore A' is virtual image of A.

A third ray incident on the mirror along AC falls normally on the mirror and retraces its path on reflection. This also appears to come from A'.

Draw A'B' perpendicular to principal axis. Therefore, A'B' is virtual image of the object AB formed after reflection at the convex mirror.

As Δ ABC and Δ A'B'C are similar, so that -

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \quad \text{—} \; (1)$$

Again as Δ ABP and A'B'P are similar, so -

$$\frac{AB}{A'B'} = \frac{PB}{PB'} \quad \text{—} \; (2)$$

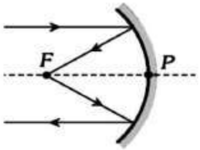
Images Formed by a Concave and Convex Mirror

Position of object	Position of Image	Size of Image	Nature of Image
Concave Mirror			
At infinity	At Focus F	Highly diminished	Real and inverted
Beyond C	Between F and C	Diminished	Real and inverted
At C	At C	Same size	Real and inverted
Between F and C	Beyond C	Enlarged	Real and inverted
At F	At infinity	Highly enlarged	Real and inverted
Between P and F	Behind the mirror	Enlarged	Virtual and erect
Convex Mirror			
At infinity	At focus	Highly diminished	Virtual point size
Anywhere on Principal axis	Between pole & focus	Diminished	Virtual and erect

Image Formation by Spherical Mirrors

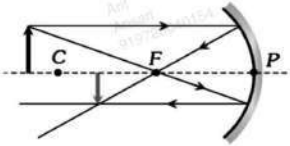
(1) When object is placed at infinite (*i.e.* $u = \infty$)

Image → At F
→ Real
→ Inverted
→ Very small in size
→ Magnification $m << -1$



(2) When object is placed between infinite and centre of curvature (*i.e.* $u > 2f$)

Image → Between F and C
→ Real
→ Inverted
→ Small in size
→ $m < -1$



(3) When object is placed at centre of curvature (*i.e.* $u = 2f$)

Image

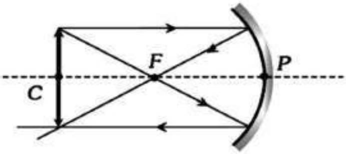
⟶ At C

⟶ Real

⟶ Inverted

⟶ Equal in size

⟶ $m = -1$



(4) When object is placed between centre of curvature and focus (*i.e.* $f < u < 2f$)

Image

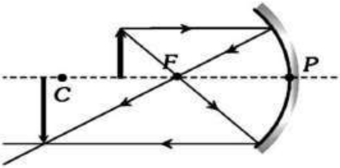
⟶ Between $2f$ and ∞

⟶ Real

⟶ Inverted

⟶ Large in size

⟶ $m > -1$



(5) When object is placed at focus (*i.e.* $u = f$)

Image

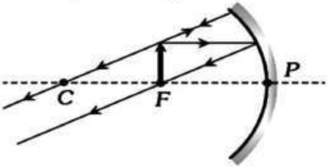
⟶ At ∞

⟶ Real

⟶ Inverted

⟶ Very large in size

⟶ $m >> -1$



(6) When object is placed between focus and pole (*i.e.* $u < f$)

Image

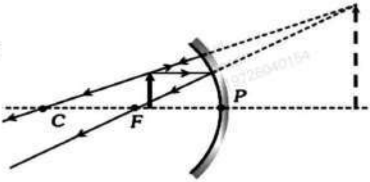
⟶ Behind the mirror

⟶ Virtual

⟶ Erect

⟶ Large in size

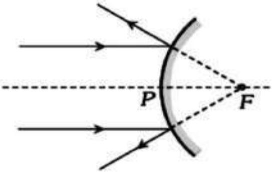
⟶ $m > +1$



Convex mirror : Image formed by convex mirror is always virtual, erect and smaller in size.

(1) When object is placed at infinite (*i.e.* $u = \infty$)

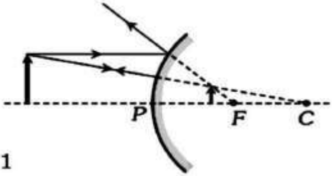
Image



- $\longrightarrow$ At F
- $\longrightarrow$ Virtual
- $\longrightarrow$ Erect
- $\longrightarrow$ Very small in size
- $\longrightarrow$ Magnification $m << +1$

(2) When object is placed any where on the principal axis

Image



- $\longrightarrow$ Between P and F
- $\longrightarrow$ Virtual
- $\longrightarrow$ Erect
- $\longrightarrow$ Small in size
- $\longrightarrow$ Magnification $m < +1$

Linear Magnification of a Spherical Mirror -

Linear magnification or simply magnification of a spherical mirror is the ratio of the size of the image formed by the mirror to the size of the object.

$$\boxed{m = \frac{\text{Size of image (I)}}{\text{Size of object (O)}} = \frac{A'B'}{AB}}$$

We assume that O is always positive. Magnification (m) will be +ve when I is +ve (i.e. image is erect) and m will be negative when I is negative (i.e. image is inverted).

For both Concave and Convex mirror we know that

$$\boxed{\frac{A'B'}{AB} = \frac{PB'}{PB}}$$

In case of Concave mirror, when image formed is real

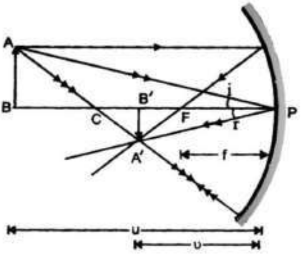
$A'B' = -I$, $AB = +O$

$PB' = -v$, $PB = -u$

$\therefore \quad \frac{-I}{O} = \frac{-v}{-u} = \frac{v}{u}$

$\therefore \quad m = -\frac{I}{O} = \frac{v}{u}$

$$\boxed{m = \frac{I}{O} = -\frac{v}{u}}$$



Note -

(i) When $m > 1$, image formed is enlarged.

(ii) When $m < 1$, image formed is diminished.

(iii) When m is positive, image must be erect (i.e. virtual)

(iv) When m is negative, image must be inverted (i.e. real)

Practical Applications of Spherical Mirrors -

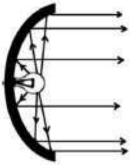
(i) A convex mirror is used as a reflector in Street Lamps. As a result, the light from the lamp diverges over a Large area.

(ii) A convex mirror is used as a drivers mirror in all vehicles like cars and scooters etc., for looking at the traffic at the rear of the vehicle. Such a mirror has a much wider field of view as compared to a plane mirror or a concave mirror. The images formed are smaller and erect.



(iii) A Concave mirror is used as a reflector in search lights, head lights of motor vehicles, telescope, solar cookers etc.

(iv) A concave mirror is also used as a sharing mirror or make up mirror as it can form erect and magnified image.



Q. An object is placed 18 cm. in front of a mirror. If the image is formed at 4 cm. to the right of the mirror, calculate its focal length. Is the mirror convex or concave? What is the nature of the image? What is the radius of curvature of the mirror?

CBSE 2017

<u>Sol</u>. Here $u = -18\text{ cm.}$, $v = 4\text{ cm.}$

from $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{f} = \frac{1}{4} - \frac{1}{18} = \frac{7}{36}$$

So $f = 36/7 = 5.14\text{ cm.}$

As focal length is positive, the mirror must be convex.

The image is virtual, erect and smaller in size.

Also $R = 2f = 2 \times 5.14 = 10.28\text{ cm.}$

[redacted]

Arif Ansari 919726040154

Q. A small candle 2.5 cm in size is placed 27 cm. in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to receive a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Sol: Size of candle (O) = 2.5 cm. , $u = -27$ cm.

$R = -36$ cm , $f = R/2 = -18$ cm.

As $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ So $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$$\frac{1}{v} = \frac{1}{-18} + \frac{1}{27} = -\frac{1}{54} \quad \text{or} \quad v = -54 \text{ cm.}$$

So the screen should be placed at 54 cm. from the mirror on the same side as the object.

If I is size of image then as the image is real.

$$m = -\frac{I}{O} = \frac{-v}{-u} = \frac{v}{u}$$

$$-\frac{I}{2.5} = \frac{(-54)}{(-27)} = 2 \Rightarrow I = -5 \text{ cm.}$$

Minus sign indicates that image is inverted.

When the candle is moved closer to mirror, the screen has to be moved away from the mirror.

However when candle is at a distance less than 18 cm from the mirror, image formed would be virtual and screen is not required, as the virtual image can not be taken on the screen.

Q. Use the mirror equation to show that -
CBSE 2015
(a) An object placed between f and 2f of a concave mirror produces a real image beyond 2f.
(b) A convex mirror always produces a virtual image independent of the location of the object.
(c) An object placed between the pole and focus of a concave mirror produces a virtual + enlarged image.

Sol. (a) for a mirror $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

for concave mirror $f < 0$ and $u < 0$

Hence $\frac{1}{-|f|} = \frac{1}{v} - \frac{1}{|u|}$ $\Rightarrow$ $\frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$

where $|2f| > |u| > |f|$

$\Rightarrow \frac{1}{v} < 0$ $\Rightarrow v < 0$

Hence image forms on the same side as that of object i.e. real image forms.

Also $|2f| > |u| > |f|$ or $\frac{1}{|2f|} < \frac{1}{|u|} < \frac{1}{|f|}$

$$\frac{1}{|2f|} - \frac{1}{|f|} < \frac{1}{|u|} - \frac{1}{|f|} < 0$$

$$-\frac{1}{|2f|} < \frac{1}{|v|} < 0 \Rightarrow v > -|2f|$$

Hence image form beyond 2f on the same side of object.

(b) for convex mirror $f > 0$ and $u < 0$

so that $\frac{1}{f} = \frac{1}{v} - \frac{1}{|u|}$ $\Rightarrow$ $\frac{1}{v} = \frac{1}{f} + \frac{1}{|u|}$

for f and $|u|$ to be positive

$\frac{1}{v} > 0$ Hence $v > 0$

Hence a virtual image of object is form.

(c) For Concave mirror $f < 0$, $u < 0$

Hence by mirror formula $-\frac{1}{|f|} = \frac{1}{v} - \frac{1}{|u|}$

$$\Rightarrow \frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$$

Here $|u| < |f|$ so that $\frac{1}{|u|} > \frac{1}{|f|}$

$\Rightarrow \frac{1}{v} > 0$ or $v > 0$

Here the image is formed on RHS of mirror i.e. virtual image.

Also by $-\frac{1}{f} = \frac{1}{|v|} - \frac{1}{|u|} \Rightarrow \frac{1}{|v|} < \frac{1}{|u|}$

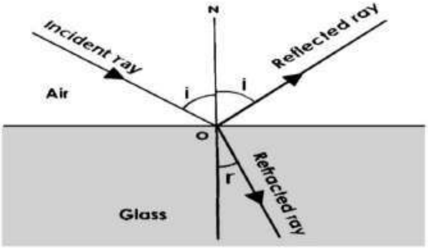
$$\Rightarrow \frac{|v|}{|u|} > 1 \quad \Rightarrow \quad m > 1$$

Hence image is enlarged.

REFRACTION

When a beam of light travelling in one transparent medium falls on another transparent medium, a part of light gets reflected back into the first medium and the rest of light enters into the other medium.

The direction of propagation of an obliquely incident ray of light that enters the other medium changes at the interface of two media. This phenomena is called refraction of light.



Refractive Index -

The ratio of speed of light in vacuum/air to the speed of light in the medium is called the refractive index (μ) of a medium.

$$\mu = \frac{\text{Speed of light in vacuum/air (c)}}{\text{speed of light in the medium (V)}}$$

When the light passes from one medium 1 to another medium 2, the refractive index of medium 2 with respect to medium 1 is written as ${}^{1}\mu_2$ and is called relative refractive index.

$$\boxed{{}^{1}\mu_2 = \frac{\mu_2}{\mu_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2}}$$

* A medium with higher refractive index is said to be optically denser compared to the medium with lower refractive index.

* Refractive index is a characteristic property of the medium, whose value depends upon nature of material of the medium and the color or wavelength of light.

Laws of Refraction -

(i) Whenever light goes from one medium to another, the frequency of light and phase of light do not change. However the velocity of light and the wavelength of light change.

(ii) The incident ray, the refracted ray and normal to the interface at the point of incidence, all lie in the same plane.

(iii) The product of refractive index and sine of angle of incidence/refraction at a point in a medium is constant.

$$\boxed{\mu \sin i = \text{Constant}} \qquad \therefore \quad \mu_1 \sin i_1 = \mu_2 \sin i_2$$

If $i_1 = i$ and $i_2 = r$, then

$$\boxed{\mu_1 \sin i = \mu_2 \sin r}$$

or $$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = {}^1\mu_2$$

where ${}^1\mu_2$ is refractive index of medium 2 with respect to medium 2.

* This Law is also called Snell's Law.

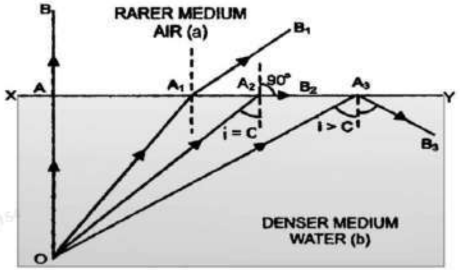
Q. CBSE 2012 For the same value of angle of incidence, the angle of refraction in three media A, B and C are 15°, 25° and 35° respectively. In which medium would the velocity of light be minimum?

Sol. From snell's Law - $\mu = \dfrac{\sin i}{\sin r} = \dfrac{c}{v}$

$\Rightarrow$ $v \propto \sin r$ for given i

Hence velocity of light is minimum in medium A as angle of refraction is minimum (15°).

Total Internal Reflection



O is a point object in the denser medium. A ray of light starting from O and incident normally along OA on XY passes straight along AB.

Another ray incident along OA_1 deviates away from normal and is refracted along A_1B_1. Clearly angle of refraction is greater than the angle of incidence and it increases with increase in the angle of incidence.

For particular value of $i = c$, the critical angle, the incident ray OA_2 is refracted at $\angle r = 90°$ and goes along the interface, along A_2B_2.

When $i > c$ as for the incident ray OA_3, the ray goes along A_3B_3, as it is reflected from interface XY. This phenomenon is called Total Internal Reflection.

Total Internal Reflection

It is the phenomenon of reflection of light into a denser medium from an interface of this denser medium and a rarer medium.

Essential Conditions for total Internal Reflection –

(i) Light should travel from a denser medium to a rarer medium.

(ii) Angle of incidence in denser medium should be greater than the critical angle for the pair of media in contact.

Critical Angle (C)

The angle of incidence in the denser medium corresponding to which angle of refraction in the rarer medium is 90°, is called critical angle for the pair of medium in contact.

It is represented by 'C' and its value depends on the nature of medium in contact.

Relation between Refractive index & Critical Angle

when $i = C$, $r = 90^\circ$

Applying Snell's Law at A_2, $\mu_b \sin C = \mu_a \sin 90^\circ$

$$\frac{\mu_b}{\mu_a} = \frac{1}{\sin C}$$

$$\boxed{{}^a\mu_b = \frac{1}{\sin C}}$$

a = rarer
b = dense

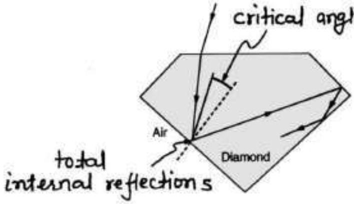
As μ depends on wavelength, therefore critical angle for the same pair of medium in contact will be different for different colors.

Applications of Total Internal Reflection

(1) The brilliance of diamond -

The brilliance of diamond is due to total internal reflection of light. μ for diamond is 2.42, so that critical angle for diamond air interface as calculated from

${}^a\mu_b = 1/\sin C$ is 24.4°

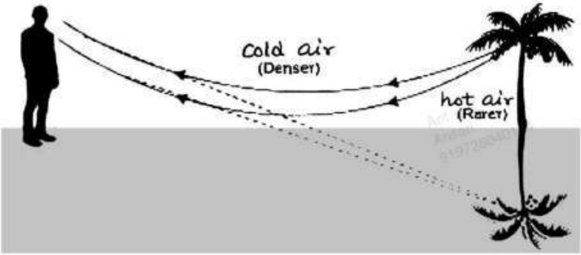


The diamond is cut suitably so that light entering the diamond from any face falls at an angle greater than 24.4°. Therefore it suffers multiple total internal reflections at various faces and remain within the diamond and does not come out. Hence the diamond sparkles.

(2) Mirage-

Mirage is an optical illusion which occurs usually in desert on hot summer days. The objects such as a tree appears to be inverted as if the tree is on the bank of a pond of water.

On a hot summer day, temperature of air near the surface of earth is maximum. The upper layers of air have gradually decreasing temperature. Therefore density and refractive index of air goes on increasing slightly with height above the surface of earth.



A ray of light from the top O of a tree goes from denser to rarer medium bending successively away from normal.

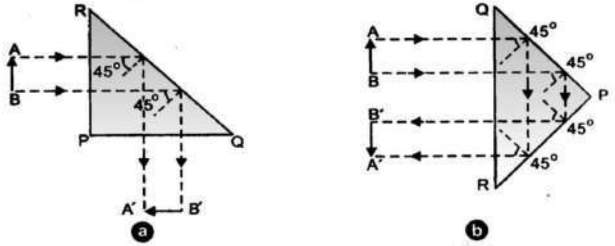
At a particular layer, when angle of incidence becomes greater than the critical angle, total internal reflection occurs and the totally reflected ray reaches the observer along AE, appearing to come from I, the mirror image of O.

Thus inverted image of tree creates the impression of reflection from a pond of water.

(3) Totally reflecting glass prisms -

These are right angled prisms which turn the light through 90° or 180°. They are based on the phenomenon of total internal reflection of light.

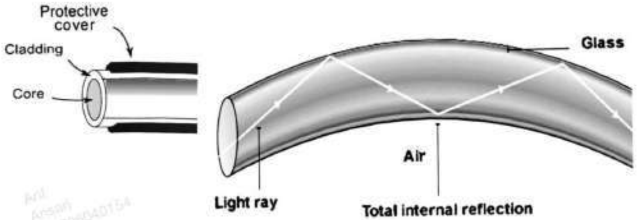
μ for glass is 1.5 so that critical angle for glass-air interface is 42°. In totally reflecting glass prisms, angle of incidence is made 45°. which is greater than the critical angle of the glass-air interface. Hence the light suffers total internal reflection.



(4) Optical Fibers -

Optical fibers consist of several thousands of very long fine quality fibers of glass or quartz, The diameter of each fiber is of the order of 10^{-4} cm with refractive index of material of the order of 1.5.

The fibers are coated with a thin layer of material ok lower refractive index of the order of 1·48.



Light incident on one end of the fiber at a small angle enters the fiber after refraction and undergoes repeated total internal reflections inside the fiber. It finally comes out of the fiber at the other end, even if the fiber is bend or twisted in any form. And there is almost no loss of light through the sides of the fiber.

Applications of optical Fibers -

(1) Optical fibers are used in transmission and reception of electrical signals by converting them first into light signals.

(2) A bundle of optical fibers is called light pipe. This pipe can transmit an image. Hence it is used in medical and optical examinations of even the inaccessible parts of human body, as in endoscopy.

(3) Optical fibers are used in telephone and other transmitting cables. Each fiber can carry upto 2000 telephone messages without much loss of intensity.

Q. Calculate the speed of light in a medium whose critical angle is 30°.
CBSE 2022

Sol. We know that $\mu = \frac{c}{v}$

Also $\mu = \frac{1}{\sin i_c}$ [i_c = Critical Angle]

$$\frac{c}{v} = \frac{1}{\sin i_c}$$

$$v = c \times \sin i_c$$

$$v = 3 \times 10^8 \times \sin 30^\circ = 1.5 \times 10^8 \, m/s$$

Spherical Refracting Surfaces -

A refracting surface which forms a part of a sphere of transparent refracting material is called a spherical refracting surface.

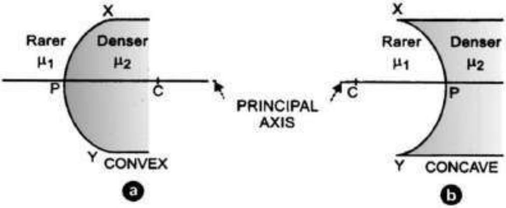
There are two types of spherical refracting surfaces-

(i) **Convex spherical refracting surface -**

It is convex towards rarer medium side.

(ii) **Concave spherical refracting surface -**

It is concave towards the rarer medium side.



Important Terms -

(i) **Pole (P)**

The centre of spherical refracting surface is called pole (P).

(ii) **Centre of curvature (C) -**

The centre of the sphere of which the spherical refracting surface is a part, is called the centre of curvature (C).

(iii) **Radius of Curvature (R) -**

Radius of the sphere of which the spherical refracting surface is a part is called radius of curvature, of the surface. It is represented by R.

(iv) Principal axis -

The line passing through the pole and the centre of curvature of spherical refracting surface, extended on either side is called Principal axis.

New Cartesian Sign Conventions -

(1) All distances are measured from the pole of the spherical refracting surface.

(2) The distances measured in the direction of incidence of light are taken as positive and the distances measured in a direction opposite to the direction of incidence of light are taken as negative.

Assumptions -

(i) The object consists only of a point laying on the principal axis of the spherical refracting surface.

(ii) The aperture of spherical refracting surface is small.

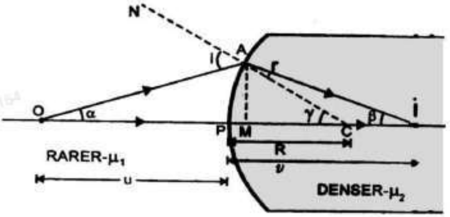
(iii) The incident and refracted rays make small angles with the principal axis of the surface.

So that -

$$\boxed{\begin{array}{l} \sin i \approx i \\ \text{and} \quad \sin r \approx r \end{array}}$$

Refraction from Rarer to Denser Medium at a convex spherical Refracting surface.

A ray of light starting from O and incident normally on the surface XY along OP passes straight.



Another ray of light incident on XY along OA at $\angle i$ is refracted along AI at $\angle r$, bending towards the normal CAN. The two refracted rays actually meet at I, which is the real image of O.

From A, draw AM perpendicular to OI.

Let $\angle AOM = \alpha$, $\angle AIM = \beta$
and $\angle ACM = \gamma$

As external angle of a triangle is equal to sum of internal opposite angles, therefore in ΔIAC

$$\left.\begin{array}{ll} \gamma = r + \beta \quad \text{or} & r = \gamma - \beta \\ \text{similarly in } \Delta OAC & i = \alpha + \gamma \end{array}\right\} \quad \text{— (1)}$$

According to snell's Law.

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\therefore \quad \mu_1 i = \mu_2 r \qquad (\because \text{angles are small})$$

Using eq. (1) we get -

$$\mu_1 (\alpha + \gamma) = \mu_2 (\gamma - \beta)$$

As angles α, β and γ are small so using $\theta \approx \tan\theta$ we get-

$$\mu_1\left(\frac{AM}{MO} + \frac{AM}{MC}\right) = \mu_2\left(\frac{AM}{MC} - \frac{AM}{MI}\right)$$

As aperture of the spherical surface is small, M is close to P. Therefore-

$$MO \approx PO \quad , \quad MI \approx PI \quad MC \approx PC$$

So
$$\mu_1\left(\frac{1}{PO} + \frac{1}{PC}\right) = \mu_2\left(\frac{1}{PC} - \frac{1}{PI}\right)$$

$$\therefore \quad \frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Using new cartesian sign conventions, we get -

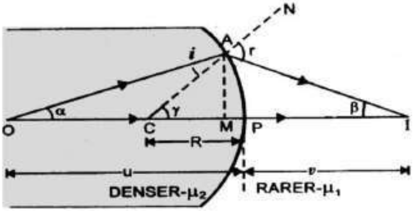
$$PO = -u \quad , \quad PI = +v \quad , \quad PC = R$$

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

This is the required relation governing refraction from rarer to denser medium at a convex spherical refracting surface.

Refraction From Denser to Rarer Medium

Let O be a point object laying on the principal axis of the spherical surface. A ray of light OA starting from O meets the refracting surface at A. On refraction it bends away from the normal CAN and moves along AI.



Another ray of light OP falling normally on the refracting surface goes undeviated along PI.

The two refracted rays AI and PI actually meet at I, which is therefore the real image of the point object O.

If i and r are the angles of incidence and refraction, then from snell's law, we have-

$$\text{or} \qquad \mu_2 \sin i = \mu_1 \sin r$$

As i and r are small so $\sin i \simeq i$ and $\sin r \simeq r$

$$\text{So} \qquad \mu_2 i = \mu_1 r$$

$$\text{In } \Delta OAC, \qquad i = \gamma - \alpha$$

$$\text{In } \Delta IAC, \qquad r = \gamma + \beta$$

$$\text{So that} \qquad \mu_2(\gamma - \alpha) = \mu_1(\gamma + \beta)$$

From A draw AM perpendicular to OI -

As angles are small, so using $\theta \simeq \tan\theta$, we get -

$$\mu_2\left[\frac{AM}{MC}-\frac{AM}{MO}\right]=\mu_1\left[\frac{AM}{MC}+\frac{AM}{MI}\right]$$

or $$\mu_2\left[\frac{1}{MC}-\frac{1}{MO}\right]=\mu_1\left[\frac{1}{MC}+\frac{1}{MI}\right]$$

since aperture of the refracting surface is small, M is close to P.

$\therefore$ $MC \approx PC$, $MO \approx PO$ and $MI \approx PI$

$$\therefore \quad \mu_2\left[\frac{1}{PC}-\frac{1}{PO}\right]=\mu_1\left[\frac{1}{PC}+\frac{1}{PI}\right]$$

Applying new cartesian sign conventions -

$PO = -u$, $PI = v$, $PC = -R$

$$\frac{\mu_2}{-u}+\frac{\mu_1}{v}=\frac{\mu_2-\mu_1}{-R}$$

$$\boxed{-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R}}$$

This is the relation governing refraction from denser to rarer medium at a convex spherical refracting surface.

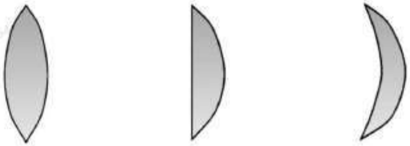
Lenses

A lense is a transparent refracting medium bound by two spherical surfaces or one spherical surface and the other plane surface.

Lenses are devided into two classes -

(i) Convex or Converging Lenses -

A convex lense is that, which is thicker in the middle and thin at the edges.



Double-Convex Lens

Plano-Convex Lens

Concavo-Convex Lens

(ii) Concave or Diverging Lenses -

A Concave Lense is thinner in the middle and thicker at the edges.

Double-Concave Lens

Plano-Concave Lens

Convexo-Concave Lens

Important terms related to Lenses -

(1) Principal axis -

It is defined as a straight line passing through the centres of curvature of two surfaces of a Lens.

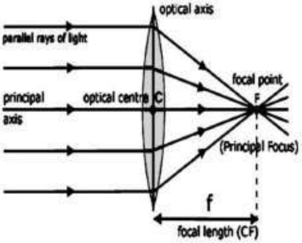
(2) Optical centre -

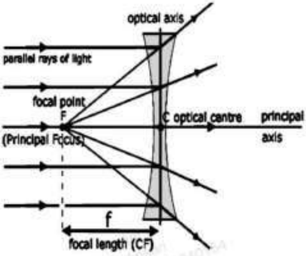
It is taken as a point lying on the principal axis so that a ray of light passing through it does not suffer any deviation from its path and it goes undeviated, without any Lateral displacement.

(3) Principal Focus -

when a beam of light is incident on a lens in a direction parallel to the principal axis of the Lens, the ray after refraction through the Lens converge to (in case of convex Lens) or appear to diverge (in case of concave Lens) from a point on the principal axis. This point F is known as the principal focus of the Lens.

CF = f is principal focal Length of the lens.





(4) Aperture -

Aperture of a lens is the effective diameter of its Light transmitting area.

Lens Maker's Formula -

It is a relation that connects focal length of a lens to radius of curvature of the two surfaces of the lens and refractive index of the material of the lens.

The lens maker's formula is useful in designing lenses of desired focal length using suitable material and surfaces of suitable radius of curvature.

New Cartesian sign Conventions -

(i) All distances are measured from the optical centre of the lens.

(ii) All the distances measured in the direction of incidence of light are taken as positive, whereas all the distances measured in a direction opposite to the direction of incidence of light are taken as negative.

(iii) For a convex lens, f is positive and for a concave lens, f is negative.

Assumptions -

(i) The lens is thin so that distances measured from the poles of its surfaces can be taken as equal to the distances from the optical centre of the lens.

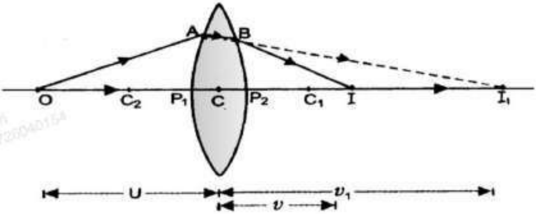
(ii) The object consists only of a point lying on the principal axis of the lens.

(iii) The aperture of the lens is small.

(iv) The incident ray and refracted ray make small angles with the principal axis of the lens.

Derivation of Lens Maker's Formula for Convex Lens -

In figure P_1, P_2 are the poles, C_1, C_2 are the centre of curvature of two surfaces of a thin convex lens XY with optical centre at C.



Consider a point lying on the principal axis of the lens. A ray of light starting from O and incident normally on the surface XP_1Y along OP_1 passes straight.

Another ray incident on XP_1Y along OA is refracted along AB. If the lens material were continuous and there were no second surface XP_2Y of the lens, the refracted ray AB would go straight meeting the first refracted ray at I_1. Therefore I_1 would have been a real image of O formed after refraction at XP_1Y.

As the refraction occurs from rarer to denser medium so we can write -

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{R}} \quad \text{—} \quad (1)$$

(where $CI_1 \approx P_1I_1 = v_1$, $CC_1 \approx P_1C_1 = R_1$, $CO \approx P_1O = -u$)

Actually, the lense material is not continuous Therefore the refracted ray AB suffers further refraction at B and emerges along BI, meeting actually the principal axis at I. Therefore I is the final real image of O, formed after refraction through the convex lens.

For refraction at the second surface XP_2Y, we can regard I_1 as a virtual object, whose real image is formed at I.

Therefore $CI_1 \approx P_2I_1 = v_1$, Let $CI \approx P_2I = v$ and let R_2 be the radius of curvature of second surface of the Lens.

As refraction is now taking place from denser to rarer medium, therefore -

$$\boxed{-\frac{\mu_2}{v_1} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R_2}} \quad \text{---} \quad (2)$$

Adding eq. (1) and (2) we get -

$$\frac{\mu_1}{-u} + \frac{\mu_1}{v} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\mu_1\left[\frac{1}{v} - \frac{1}{u}\right] = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Put $\frac{\mu_2}{\mu_1} = {}^1\mu_2$ = refractive index of material with respect to surrounding material

When object on the left of lens is at infinity, image is formed at the principal focus of the lens.

$\therefore$ when $u = \infty$, $v = f$ = focal length of the lens.

then $$\frac{1}{f} - \frac{1}{\infty} = ({}^1\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\boxed{\frac{1}{f} = ({}^1\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Lens Formula

It is a relation between focal length of a lens and distances of object and image from optical centre of the lens.

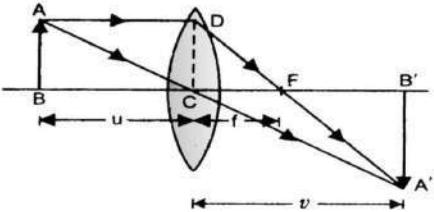
(A) Convex Lens -

The image formed may be real or virtual depending on the position of the object.

(i) Real Image -

AB is an object held perpendicular to the principal axis of the lens at a distance beyond focal length of the lens.

A real, magnified and inverted image A'B' is formed.



As $\Delta A'B'C$ and ΔABC are similar, so

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \text{—— (1)}$$

again $\Delta A'B'F$ and ΔCDF are similar, so -

$$\frac{A'B'}{CD} = \frac{F'B'}{CF}$$

But $CD = AB$ So - $\dfrac{A'B'}{AB} = \dfrac{FB'}{CF}$ —— (2)

From eq. (1) and (2) we get-

$$\frac{CB'}{CB} = \frac{FB'}{CF} = \frac{CB' - CF}{CF}$$

Using new Cartesian sign Conventions -

Let $CB = -u$, $CB' = +v$, $CF = +f$

$$\therefore \quad \frac{v}{-u} = \frac{v-f}{f}$$

$$vf = -uv + uf$$

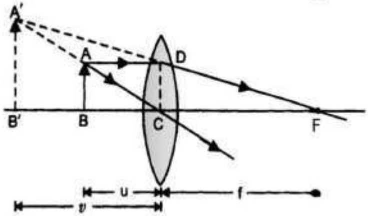
$$\text{or} \quad uv = uf - vf$$

Divide both sides by uvf, we get -

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

(ii) <u>Virtual Image</u> -

When the object AB is held close to the lens, between C and F, the image A'B' formed by convex lens is virtual, erect and magnified.



As $\Delta A'B'C$ and ΔABC are similar, so that -

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \text{--- (1)}$$

Again as $\Delta A'B'F$ and ΔCDF are similar, so that.

$$\frac{A'B'}{CD} = \frac{B'F}{CF}$$

But $CD = AB$ so -

$$\frac{A'B'}{AB} = \frac{B'F}{CF} \quad \text{--- (2)}$$

from eq. 1 & 2 we get -

$$\frac{CB'}{CB} = \frac{B'F}{CF} = \frac{CB' + CF}{CF}$$

Using new sign conventions, we put -

$CB = -u$, $CB' = -v$, $CF = +f$

$$\therefore \quad \frac{-v}{-u} = \frac{-v+f}{f} \quad \text{or} \quad uv = uf - vf$$

deviding both sides by uvf, we get -

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

* Lens formula is the same whether image is real or virtual.

Q. A convex lens of refractive index 1.5 has a focal length of 18 cm. in air. calculate the change in its focal length when it is immersed in water of refractive index 4/3.
CBSE 2008

Sol. Give that - ${}^{a}\mu_g = 1.5$, ${}^{a}\mu_w = 4/3$

$f_1 = 18$ cm.

Now $${}^{w}\mu_g = \frac{{}^{a}\mu_g}{{}^{a}\mu_w} = \frac{1.5}{4/3} = \frac{4.5}{4}$$

As $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \quad \frac{f_2}{f_1} = \frac{({}^{a}\mu_g - 1)}{({}^{w}\mu_g - 1)} = \frac{(1.5 - 1)}{\left(\frac{4.5}{4} - 1\right)}$$

$$\frac{f_2}{f_1} = 4 \quad \Rightarrow f_2 = 4f_1$$

$$f_2 = 4 \times 18 = 72 \text{ cm.}$$

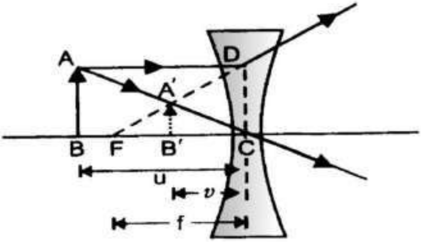
Change in focal length $= 72 - 18 = 54$ cm.

(B) Concave Lens -

In this case, the image formed is always virtual, whatever be the position of the object.

Let C be the optical centre and F be the principal focus of a concave lens of focal length f.

AB is an object held perpendicular to the principal axis of the lens. A virtual, erect and smaller image A'B' is formed due to refraction through concave lens.



As $\Delta A'B'C$ and ΔABC are similar, so that-

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \text{--- (1)}$$

Again as $\Delta A'B'F$ and ΔCDF are similar-

$$\therefore \quad \frac{A'B'}{CD} = \frac{B'F}{CF}$$

But $CD = AB$, so that $\dfrac{A'B'}{AB} = \dfrac{B'F}{CF}$ --- (2)

from equation (1) and (2)

$$\frac{CB'}{CB} = \frac{B'F}{CF} = \frac{CF - CB'}{CF}$$

Using New Cartesian Sign Conventions, we get-

$$CB = -u \quad , \quad CB' = -v \quad , \quad CF = -f$$

$$-\frac{v}{u} = \frac{-f+v}{-f}$$

$$vf = uf - uv$$

$$uv = uf - vf$$

devide both sides by uvf, we get

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

It is the required Lens formula

Q. (CBSE 2007) A double convex lens of glass of refractive index 1.5 has its both surfaces of equal radius of curvature of 30 cm each. An object of height 5 cm. is placed at a distance of 12.5 cm. from the lens. Calculate the size of the image formed.

Sol. Given that - $R_1 = 30\,cm$, $R_2 = -30\,cm$, $\mu = 1.5$

Using Lens maker's formula

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{30} + \frac{1}{30}\right)$$

$$\Rightarrow \frac{1}{f} = \frac{0.5}{15} \qquad \Rightarrow f = 30\,cm.$$

Given, $u = -12.5\,cm$

$\therefore$ Using lens equation $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \frac{1}{30} = \frac{1}{v} + \frac{1}{12.5} \qquad \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{12.5}$$

$$\Rightarrow \frac{1}{v} = \frac{12.5-30}{12.5\times 30} = -\frac{7}{150}$$

$$\Rightarrow v = -\frac{150}{7}\,cm.$$

But magnification $(m) = \frac{I}{O} = \frac{v}{u}$

or $$I = \frac{v}{u}\times O = \frac{(-150/7)}{-12.5}\times 5$$

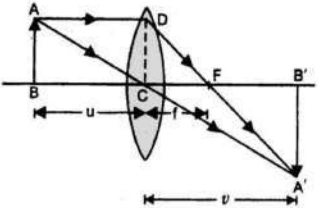
$$I = \frac{60}{7} = 8.55\,cm.$$

Linear Magnification Produced by a Lens (m)-

The linear magnification produced by a Lens is defined as the ratio of the size of the image (I) to the size of the object (O).

$$\boxed{m = \frac{\text{Size of Image } (A'B')}{\text{Size of object } (AB)} = \frac{I}{O}}$$

In a convex lens, when image formed is real, as shown in figure, then-



$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

$$m = \frac{-I}{O} = \frac{+V}{-u}$$

or $$\boxed{m = \frac{I}{O} = \frac{V}{u}}$$

We assume that O is always positive.
Magnification (m) will be +ve when I is +ve (i.e. image is erect) and m will be negative when I is negative (i.e. image is inverted).

Note-

(i) When $m > 1$, image formed is enlarged.

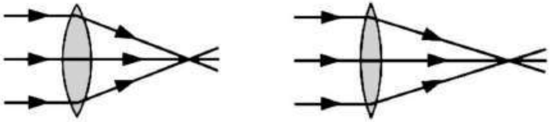
(ii) When $m < 1$, image formed is diminished.

(iii) When m is positive, image must be erect (i.e. virtual)

(iv) When m is negative, image must be inverted (i.e. real)

Power of a Lens (P)

The ability of the lens to converge a beam of light falling on the lens is called power of Lens.



It is measured as the reciprocal of focal length of the lens.

$$\boxed{P = \frac{1}{f}}$$

According to lens maker's formula -

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\boxed{P = ({}^{1}\mu_{2}-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

For a converging lens, power is positive and for a diverging lens, power is negative.

* The SI unit of power is dioptre (D).

1 dioptre -

It is the power of a lens of focal length one meter.

* when f is in cm.

$$\boxed{P = \frac{1}{f/100} = \frac{100}{f}}$$

Q. Find the radius of curvature of convex surface of a plano convex lens, whose focal length is 0.3m and $\mu = 1.5$.

CBSE 2018

Sol. $R_1 = \infty$, $R_2 = ?$, $f = 0.3m$, $\mu = 1.5$

from $\frac{1}{f} = ({}^{a}\mu_{g}-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{0.3} = (1.5-1)\left(\frac{1}{\infty} - \frac{1}{R_2}\right)$$

$$-\frac{1}{R_2} = \frac{1}{0.3 \times 0.5} = \frac{100}{15}$$

$$R_2 = -0.15\ m.$$

Q. A convex lens made up of glass of refractive index 1.5 is dipped in turn, in

CBSE 2011

(a) a medium of refractive index 1.65

(b) a medium of refractive index 1.33

will it behave as a converging or a diverging lens in the two cases?

Sol. From Lens maker's formula.

$$\frac{1}{f} = ({}^{m}\mu_{g} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\frac{1}{f} = \left(\frac{{}^{a}\mu_{g}}{{}^{a}\mu_{m}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As ${}^{a}\mu_{g} < {}^{a}\mu_{m}$ for the first medium of refractive index 1.65.

And ${}^{a}\mu_{g} > {}^{a}\mu_{m}$ for the second medium of refractive index 1.33.

Hence, the value of focal length f will be negative in the first medium, so the convex lens behaves as the diverging lens for the first medium.

The value of focal length f will be positive in the second medium, so the convex lens behaves as the converging lens for second medium.

Q. A beam of light converges at a point P. A concave lens of focal length 16 cm. is placed in the path of this beam 12 cm. from P. Draw a ray diagram and find the location of the point at which the beam would now converge.

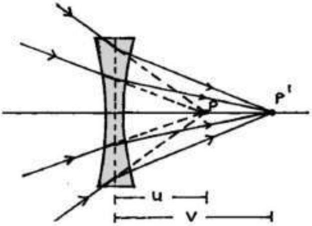
CBSE 2017

Sol.

$u = 12$ cm

$v = +48$ cm.

$f = -16$ cm.



Using lens formula - $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

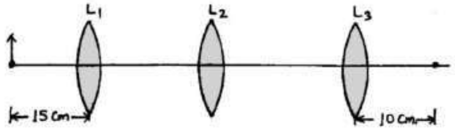
$$-\frac{1}{16} = \frac{1}{v} - \frac{1}{12} \Rightarrow \frac{1}{v} = \frac{1}{12} - \frac{1}{16}$$

$$\frac{1}{v} = \frac{1}{48} \Rightarrow v = +48 \text{ cm.}$$

The image of virtual object at P forms at P' which is at a distance 48 cm from the lens.

Q. You are given three Lenses L_1, L_2 and L_3 each of focal length 10 cm. An object is kept at 15 cm in front of L_1 as shown. The final real image is formed at the focus of L_3. Find the separation between L_1, L_2 and L_3.

CBSE 2018



Sol. For Lens L_1 $\qquad \frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$

Here $u = -15$ cm and $f = +10$ cm so that

$$\frac{1}{10} = \frac{1}{v_1} + \frac{1}{15} \qquad \Rightarrow \qquad v_1 = 30 \text{ cm}$$

Hence Distance of image from L_1 is $v_1 = 30$ cm.

For lens L_3 $\qquad \frac{1}{f_3} = \frac{1}{v_3} - \frac{1}{u_3}$

Here $v_3 = +10$ cm and $f_3 = +10$ cm.

so that $\qquad \frac{1}{10} = \frac{1}{10} - \frac{1}{u_3}$

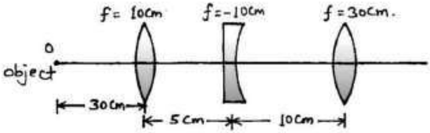
$$\frac{1}{u_3} = 0 \qquad \Rightarrow \qquad u_3 = \infty$$

The refracted rays from Lens L_2 becomes parallel to principal axis. It is possible only when image formed by L_1 lies at first focus of L_2 i.e. at a distance of 10 cm from L_2.

$\therefore$ separation between L_1 and L_2 = 30 + 10 = 40 cm.

The distance between L_2 and L_3 may take any value.

Q. CBSE 2011 Find the position of the image formed of the object O by the lens combination given in the figure.



Sol. For lens of focal length 10 cm - $f = +10cm$, $u = -30cm$.
Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{10} = \frac{1}{v} - \frac{1}{(-30)}$$

$$\frac{1}{v} = \frac{1}{15} \Rightarrow v = 15\ cm$$

The image formed by first lens acts as an virtual object for plano-concave lens.

For plano-concave lens -

$$u = 10cm \quad , \quad f = -10\ cm$$

Hence by using lens formula -

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow -\frac{1}{10} = \frac{1}{v} - \frac{1}{10}$$

$$\frac{1}{v} = 0 \Rightarrow v = \infty$$

The refracted ray becomes parallel to principal axis for convex lens of focal length 30 cm.

For the second convex lens -

$$u = -\infty \quad , \quad f = 30\ cm.$$

by using the lens formula -

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{30} = \frac{1}{v} - \frac{1}{(-\infty)}$$

$$\frac{1}{v} = \frac{1}{30} \Rightarrow v = 30\ cm.$$

So the final image forms at a distance of 30cm. from the second convex lens on the other side of it.

Combination of Lenses in contact -

In various optical instruments, two or more lenses are combined to

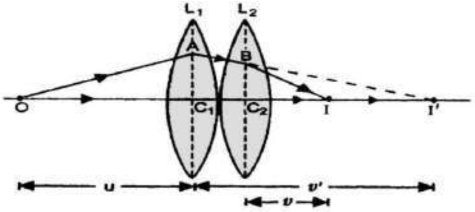
(i) Increase the magnification of the image.
(ii) make the final image erect w.r.t. the object.
(iii) reduce certain aberrations (i.e. defects of images by single lens)

To obtain the position, size, and nature of the final image produced by a combination of Lenses, we first find the image of the object formed by the first lens.

This image acts as an object for the second lens and we locate its image. The image formed by the second lens serves as the object for the third lens and so on.

Focal length of Equivalent Lens -

(a) Both Lenses are Convex -



Let a point object O be placed on the common principal axis at a distance $OC_1 = u$. The Lens L_1 alone would form its image at I' where $C_1I' = v'$.

From the lens formula -

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \qquad \text{--- (1)}$$

I' would serve as a virtual object for lens L_2, which forms a final image I at distance $C_2I = v$.

As the Lenses are thin, therefore, for the lens L_2

$$u = C_2I' \simeq C_1I' = v'$$

from the lens formula for L_2 -

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \qquad — (2)$$

Adding eq. (1) and (2), we get-

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \qquad — (3)$$

Let the two lenses be replaced by a single lens of focal length F, which forms image I at distance v of an object at distance u from the lens.
For this Lens-

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F} \qquad — (4)$$

from eq. (3) and (4), we get-

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

(b) One lens is convex and the other is concave

If f_1 is the focal length of convex lens and f_2 is focal length of Concave Lens, then their equivalent focal length F would be given as-

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{-f_2} = \frac{1}{f_1} - \frac{1}{f_2}$$

There are three cases arise-

Case 1:

If $f_1 = f_2$, then $F = \infty$.
The combination would behav like a plane glass plate.

Case 2:

If $f_1 > f_2$, then F is negative.
Therefore the combination behave as a concave lens, when focal length of convex lens is Larger.

Case 3: If $f_1 < f_2$ then F is positive.

Therefore the combination would behave as a convex Lens, when focal length of convex lens is smaller.

Note-

(i) If the lenses of focal lengths f_1 and f_2 are separated by a finite distance d, the focal length F of the equivalent lense is given by -

$$\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

(ii) Total Power of any number of Lenses in contact is equal to the algebraic sum of the powers of individual lenses.

$$\boxed{P = P_1 + P_2 + \dots P_n}$$

★ If the sum of the terms on right side is positive, the system of lenses would behaves like convex.

★ If the sum of the terms on right side is negative, the system of lenses would behave as concave.

Q. (CBSE 2021) Two thin lenses of power −4D and 2D are placed in contact. Find the focal length of the combination.

Sol- Net power of the lenses $P = P_1 + P_2$

$$P = -4 + 2 = -2\ D$$

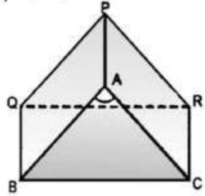
focal length $f = \frac{1}{P}$

$$f = \frac{1}{-2} = -0.5\ m$$

Prism

A prism is a portion of a transparent medium bounded by two plane faces inclined to each other at a suitable angle.

In figure ABQP and ACRP are the two refracting faces. Angle A between them is the refracting angle or angle of prism.

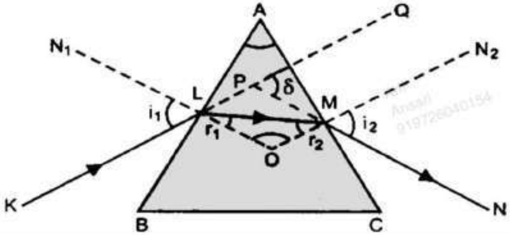


A section ABC of the prism made by a plane at right angles to the refracting edge of the prism is called Principal section of the prism.

(i) Calculation of angle of deviation -

In figure ABC is principal section of a prism with angle of prism = A.

A ray of light KL is incident on the face AB of the prism at $\angle i_1$. It is refracted along LM at $\angle r_1$ bending towards the normal N_1O.



The refracted ray LM is incident at $\angle r_2$ on face AC of the prism. It bends away from normal N_2O and emerges along MN at $\angle i_2$.

In passing through the prism ray KL suffers two refractions and has turned through an $\angle QPN = \delta$, which is the angle of deviation.

In ΔPLM, $\delta = \angle PLM + \angle PML$

$$\delta = (i_1 - r_1) + (i_2 - r_2)$$

$$\delta = (i_1 + i_2) - (r_1 + r_2) \quad \text{--- (1)}$$

In ΔOLM, $\angle O + r_1 + r_2 = 180^\circ \quad \text{--- (2)}$

In quadrilateral ALOM,

As $\angle L + \angle M = 180^\circ$ ($\because$ each angle is 90°)

$\therefore$ $A + \angle O = 180^\circ$

Using equation (2), $\angle O + r_1 + r_2 = A + \angle O$

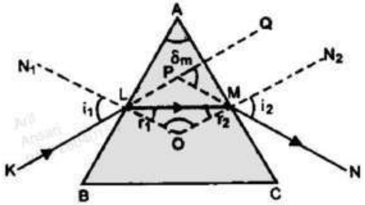
$$\boxed{r_1 + r_2 = A} \quad \text{--- (3)}$$

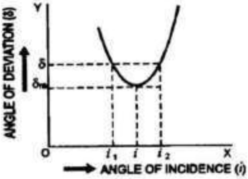
Put in eq. (1) $\boxed{\delta = (i_1 + i_2) - A} \quad \text{--- (4)}$

Prism formula

In the minimum deviation position, $i_1 = i_2 = i$

As $i_1 = i_2$, therefore $r_1 = r_2 = r$





then from equation ③, $r_1 + r_2 = A$

$$r + r = A \qquad \therefore \quad \boxed{r = \frac{A}{2}}$$

From eq. ④, $\delta_m = i + i - A$

or $A + \delta_m = 2i$ or $\boxed{i = \frac{A + \delta_m}{2}}$

If μ is refractive index of the prism w.r.t. air then by snell's Law,

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \boxed{\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2}}$$

This relation is called **Prism formula**.

* It is used for accurate determination of refractive index of a transparent medium of which the prism is made.

Q. A ray of light passing through an equilateral triangular glass from air undergoes minimum deviation when angle of incidence is 3/4 of the angle of prism. Calculate the speed of light in the prism.

Delhi 2018

Sol. When prism is adjusted at angle of minimum deviation then-

$$\mu = \frac{\sin i}{\sin r}$$

Where $r = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$

and $i = \frac{3}{4}A = \frac{3}{4} \times 60 = 45^\circ$

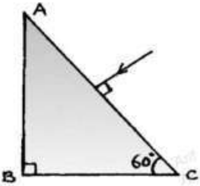
$$\Rightarrow \quad \mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$$

But refractive index $\mu = \frac{c}{v}$

$$v = \frac{c}{\mu} \Rightarrow \frac{3 \times 10^8}{\sqrt{2}} = 2.1 \times 10^8 \text{ m/s}$$

Q. Trace the path of a ray of light passing through a glass prism ABC as shown in figure. If the refractive index of the glass is $\sqrt{3}$. Find out the value of the angle of emergence from the prism.

CBSE 2012

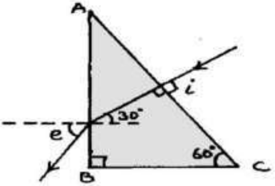


Sol.

Since $i = 0$, At the interface AC, we have according to snell's Law -

$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_a}$$

Given that -
$\mu_g = \sqrt{3}$



$$\sin r = \frac{\mu_a}{\mu_g} \sin i = 0 \qquad (\text{as } i = 0)$$

Hence $\quad r = 0$

This ray pass unrefracted at AC interface and reaches AB interface. By the diagram we can see angle of incidence becomes 30°.

Thus applying snell's law -

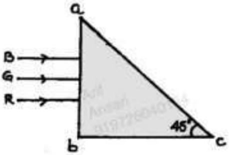
$$\frac{\sin 30^\circ}{\sin e} = \frac{\mu_a}{\mu_g} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sin e = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}$$

Thus $\quad e = 60^\circ$ (angle of emergence)

Q. Three light rays red (R), green (G) and blue (B) are incident on a right angled prism abc at face ab. The refractive indices of the material of the prism for red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. Out of the three which colour ray will emerge out of face ac? Trace the path of these rays after passing through face ab.

Delhi 2019



sol.

By geometry, angle of incidence (i) of all three rays is 45° at face ac.

Light suffer total internal reflection for which this angle of incidence is greater than critical angle.

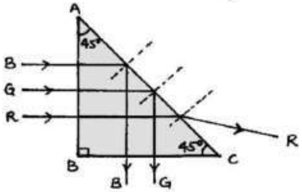
$$i > i_c$$

$$\Rightarrow \sin i > \sin i_c$$

$$\sin 45^\circ > \sin i_c$$

$$\frac{1}{\sin 45^\circ} < \frac{1}{\sin i_c}$$

Hence $\sqrt{2} < \mu$

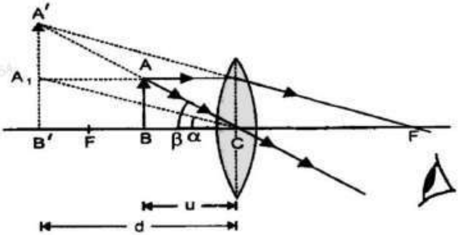


Total internal reflection takes place on ac for rays $\mu > \sqrt{2} = 1.414$.

Hence green and blue suffer total internal reflection whereas red undergo refraction, and red colour ray will emerge out of face ac.

Simple Microscope or Magnifying Glass-

A simple microscope consist of converging lens of small focal length. A virtual, erect and magnified image of the object is formed at the least distance of distinct vision from the eye held close to the lens.



Let an object AB is held between optical centre C and principal focus F of the lens perpendicular to the principal axis. A virtual, erect and magnified image A'B' is formed as traced in fig. The eye is held close to the lens, and CB' = d (= 25 cm.)

Magnifying power of a simple microscope is defined is defined as the ratio of the angles subtende by the image and the object on the eye, when both are at the least distance of distinct vision from eye

Let $\angle A'CB' = \beta$ and $\angle A_1CB' = \alpha$.

By definition, Magnifying Power $(m) = \frac{\beta}{\alpha}$

for small angles $\tan\theta \simeq \theta$

Hence $\tan\alpha = \alpha$ and $\tan\beta = \beta$

$$\therefore \quad m = \frac{\tan\beta}{\tan\alpha}$$

In ΔABC $\tan\beta = \frac{AB}{CB}$

In $\Delta A_1B'C$ $\tan\alpha = \frac{A_1B'}{CB'} = \frac{AB}{CB'}$

Hence $m = \frac{AB}{CB} \times \frac{CB'}{AB} = \frac{CB'}{CB} = \frac{-v}{-u} = \frac{v}{u}$

From lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Multiply both sides by v $\frac{v}{f} = 1 - \frac{v}{u}$

or $\frac{v}{f} = 1 - m$ or $m = 1 - \frac{v}{f}$

but $v = -d$ $\therefore$ $\boxed{m = \left(1 + \frac{d}{f}\right)}$

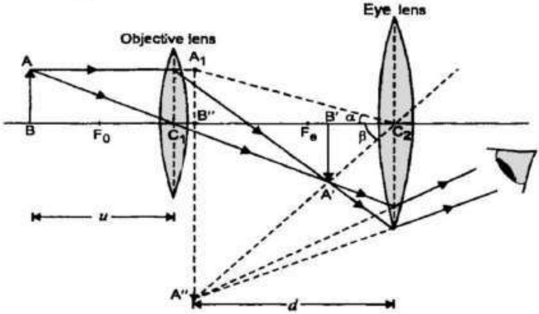
This the expression for magnifying power of a simple microscope (magnifying glass).

★ As f decreases, m increases i.e. smaller the focal length of the lens, greater is its magnifying power.

Compound Microscope-

A compound microscope consist of two converging lenses an objective lens O of very small focal length and short aperture and an eye piece E of moderate focal length and large aperture.

The two lenses are held co-axially at the free ends of a tube at a suitable fixed distance from each other. The distance of the objective lens from the object can be adjusted by rack and pinion arrangement.



Let AB is a tiny object held perpendicular to the principal axis in front of the objective lens beyond its principal focus F_0. A real, inverted and enlarged image A'B' of this object is formed by the objective lens.

Now A'B' acts as an object for the eye lens, whose position is so adjusted that A'B' lies between optical centre C_2 of eye lens and its principal focus F_e. A virtual and magnified image A''B'' is formed by the eye lens.

This image is erect w.r.t. $A'B'$ but inverted w.r.t. AB. The final image $A''B''$ is seen by the eye held close to eye lens. The adjustments are so made that $A''B''$ is at the least distance of distinct vision (d) from the eye. i.e. $C_2B'' = d$.

Magnifying power of a compound microscope is defined as the ratio of the angle subtended at the eye by the final image to the angle subtended at the eye by the object, when both the final image and the object are situated at the least distance of distinct vision from the eye.

$$\text{Magnifying power } (m) = \frac{\beta}{\alpha}$$

for small angles, $\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$\therefore \quad m = \frac{\tan\beta}{\tan\alpha}$$

In $\Delta A''B''C_2$, $\tan\beta = \dfrac{A''B''}{C_2B''}$

In $\Delta A_1B''C_2$, $\tan\alpha = \dfrac{A_1B''}{C_2B''} = \dfrac{AB}{C_2B''}$

Hence $m = \dfrac{A''B''}{C_2B''} \times \dfrac{C_2B''}{AB} = \dfrac{A''B''}{AB}$

or we can write- $m = \dfrac{A''B''}{A'B'} \times \dfrac{A'B'}{AB}$

$$\boxed{m = m_e \times m_o}$$

where $m_e = \dfrac{A''B''}{A'B'}$ = magnification by eye lens.

and $m_o = \dfrac{A'B'}{AB}$ = magnification by objective lens.

Now we know that - $m_e = \left(1 + \dfrac{d}{f_e}\right)$

Also $m_o = \dfrac{A'B'}{AB} = \dfrac{C_1B'}{C_1B} = \dfrac{V_o}{-u_o}$

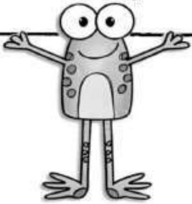
Hence Magnifying power $\boxed{m = -\frac{v_o}{u_o}\left(1 + \frac{d}{f_e}\right)}$

length of microscope tube $\boxed{L = v_o + u_e}$

Note-

(i) As magnifying power (m) is negative, the image seen in a microscope is always inverted.

(ii) For large magnifying power f_o and f_e both have to be small. Also f_o is taken to the smaller than f_e so that field of view may be increased.



Q. A compound microscope uses an objective lens of focal length 4 cm. and eyepiece lens of focal length 10 cm. An object is placed at 6 cm. from the objective lens. Calculate the magnifying power of the compound microscope. Also calculate the length of the microscope.

CBSE 2017

Sol. Given that -

$f_o = 4$ cm. $f_e = 10$ cm, $u_o = -6$ cm,

$V_e = D = 25$ cm.

For objective lens - $\frac{1}{f_o} = \frac{1}{V_o} - \frac{1}{u_o}$

$\Rightarrow \quad \frac{1}{4} = \frac{1}{V_o} + \frac{1}{6} \quad \Rightarrow \quad V_o = 12$ cm.

Magnifying power (m) $= -\frac{V_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

$m = -\frac{12}{6}\left(1 + \frac{25}{10}\right) = -7$

Length of microscope $= |V_o| + |u_e|$

for eye lens $\Rightarrow$ $V_e = -25$ cm, $f_e = 10$ cm.

by lens formula $\quad \frac{1}{f_e} = \frac{1}{V_e} - \frac{1}{u_e}$

$\frac{1}{u_e} = \frac{1}{V_e} - \frac{1}{f_e} \quad \Rightarrow \quad \frac{1}{u_e} = \frac{1}{-25} - \frac{1}{10}$

$u_e = -\frac{50}{7} \quad = -7.14$ cm.

$\therefore$ Length of microscope $= 12 + 7.14$

$L = 19.14$ cm

Astronomical Telescope 

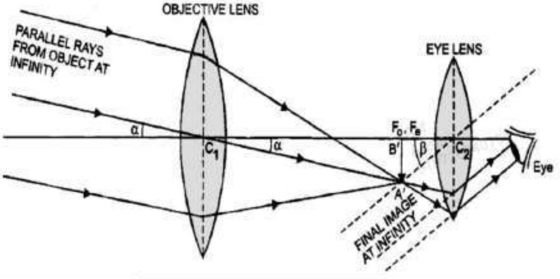
An astronomical telescope is an optical instrument which is used for observing distinct images of heavenly bodies like stars, planets etc.

It consist of two lenses, the objective lens which is of large focal length and large aperture and the eye lens, which has a small focal length and small aperture.

(i) Normal Adjustment -

Let a parallel beam of light from an astronomical object (at infinity) is made to fall on the objective lens of the telescope. It forms a real, inverted and diminished image A'B' of the object.

The eye piece is so adjusted that A'B' lies just at the focus of the eye piece. Therefore a final highly magnified image is formed at infinity The final image is erect w.r.t. A'B' and is inverte w.r.t. the object.



Magnifying Power of an astronomical telescope in normal adjustment is defined as the ratio of the angle subtended at the eye by the final image to the angle subtended at the eye, by the object directly, when the final image and the object both lie at infinite distance from the eye.

As the object lies at very huge distance, therefore angle subtended by the object at C_2 is almost the same as the angle subtended by the object at C_1 (because C_1 is close to C_2). Let it be α i.e. $\angle A'C_1B' = \alpha$.

Rays coming from the final image at infinity make $\angle A'C_2B' = \beta$ on the eye. Therefore.

Magnifying power $m = \frac{\beta}{\alpha}$

As angles are small - $\alpha = \tan\alpha$, $\beta = \tan\beta$

$$m = \frac{\tan\beta}{\tan\alpha}$$

In $\Delta A'B'C_2$, $\tan\beta = A'B'/C_2B'$

In $\Delta A'B'C_1$, $\tan\alpha = A'B'/C_1B'$

Hence $m = \frac{A'B'}{C_2B'} \times \frac{C_1B'}{A'B'} = \frac{C_1B'}{C_2B'}$

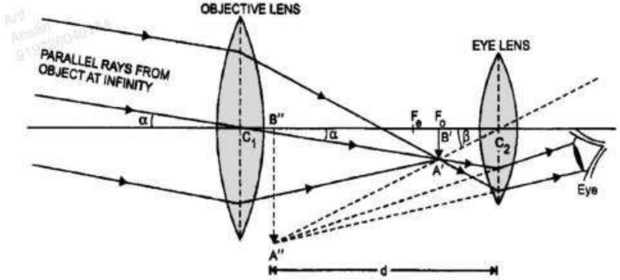
or $\boxed{\text{Magnifying Power } (m) = \frac{f_o}{-f_e}}$

* Negative sign of m indicates that final image is inverted.

length of the telescope $\boxed{L = f_o + f_e}$

(ii) When the final Image is formed at the least distance of distinct vision (d) from the eye-

Magnifying Power of an astronomical telescope is defined as the ratio of the angle subtended at the eye by the final image at the least distance of distinct vision to the angle subtended at the eye by the object at infinity, when seen directly.



As the object is very far off and C_1, C_2 are close by, angle subtended by the object on the eye at C_2 is almost the same as the angle subtended by it at C_1. Let it be α.

$$\therefore \quad \angle A'C_1B' = \alpha$$

and let $\angle A''C_2B'' = \beta$, where $C_2B'' = d$

$$\therefore \text{ Magnifying Power } (m) = \frac{\beta}{\alpha}$$

As angles α and β are small, therefore -

$$m = \frac{\tan\beta}{\tan\alpha}$$

In $\triangle A'B'C_2$, $\quad \tan\beta = \dfrac{A'B'}{C_2B'}$

In $\Delta A'B'C_1$, $\tan\alpha = \dfrac{A'B'}{C_1B'}$

So that - $m = \dfrac{A'B'}{C_2B'} \times \dfrac{C_1B'}{A'B'} = \dfrac{C_1B'}{C_2B'} = \dfrac{f_o}{-u_e}$

where f_o = focal length of objective lens and u_e = distance of $A'B'$, acting as the object for eye lens.

For eye lens $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

or $\dfrac{1}{-d} - \dfrac{1}{-u_e} = \dfrac{1}{f_e}$

or $\dfrac{1}{u_e} = \dfrac{1}{f_e} + \dfrac{1}{d} = \dfrac{1}{f_e}\left(1 + \dfrac{f_e}{d}\right)$

Hence $\boxed{m = -\dfrac{f_o}{f_e}\left(1 + \dfrac{f_e}{d}\right)}$

★ Negative sign of magnifying power indicates that final image in an telescope is inverted.

length of the telescope $\boxed{L = f_o + u_e}$

★ For large magnifying power, f_o must be as large as possible and f_e must be as small as possible. Hence in telescope aperture of objective lens is made large to increase magnifying power and resolving power of the telescope.

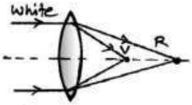
Hence $\boxed{m_{min} = -\dfrac{f_o}{f_e}}$ $\boxed{m_{max} = -\dfrac{f_o}{f_e}\left(1 + \dfrac{f_e}{d}\right)}$

Chromatic Aberration -

When an image of white object is formed by a lens then the lens will not focus different Colours at exactly the same place because the focal length of lens depends upon the refractive index of the material of the lens.

The refractive index of glass is different for different wavelengths of light.

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



As $\mu_r < \mu_v$ so that $f_r > f_v$

Correction - One way to minimize chromatic aberration is to use two or more lenses in combination so that the images of all the colours are formed at the same position.

Spherical Aberration -

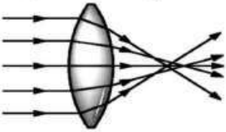
It is the defect of lens due to which the parallel light rays of incoming light do not converge at the same point after passing through the lens.

Spherical aberration can affect resolution and clarity, making it hard to obtain sharp images.

It is due to that the light rays passing through a lens near its horizontal axis are refracted less than the rays closer to the edge or "periphery", of the lens.

Correction -

This defect can be reduced by using Lenses of large focal length and small aperture.

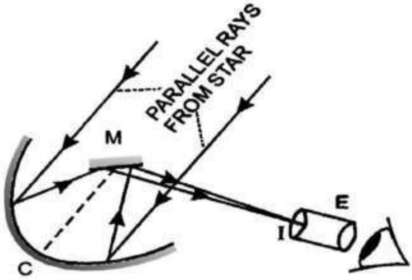


Newtonian reflecting type Telescope

The parallel beam of light coming from the distant star is reflected by large parabolic concave reflector C on to a plane mirror M. This mirror is inclined at an angle of 45° to the axis of C.

The plane mirror reflects the beam forming a real image I in front of an eye piece E.

The eye piece acts as a magnifier and the final virtual and magnified image of the star is seen distinctly by the eye.



Advantages-

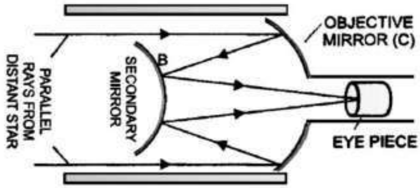
(i) There is no chromatic and spherical aberration due to use of mirror objective in place of lenses.

(ii) Image is brighter as compared to refracting type telescope and high resolution is acheived by using a mirror of Large aperture.

Reflecting type Telescope (Cassegrainian Telescope)

It is an improvement over the refracting type telescope, objective lens is replaced by a concave parabolic mirror of large aperture, which is free from chromatic and spherical aberrations.

The image formed is much brighter and the reflecting type telescope has much higher resolving power compared to the refracting type telescope. Such a telescope is known as Cassegrainian telescope on the name of its inventor.



Parallel rays from a distant star entering the telescope in a direction parallel to principal axis of the mirror tend to collect at the focus of the mirror But these reflected rays encounter a secondary convex mirror B before meeting at the focus. The convex mirror reflects them onto the eye piece, and the final image is seen through the eye piece.

The final image as seen by the eye piece is inverted w.r.t. the object.

In normal adjustment, magnifying power of a reflecting type telescope is given by-

$$\boxed{m = \frac{f_0}{f_e} = \frac{R/2}{f_e}}$$

where R is radius of curvature of concave reflector.

Q. (CBSE 2020) An astronomical telescope uses two lenses of power 10 D and 1 D. What is the magnifying power in the normal adjustment?

Sol. As $f_e = \frac{1}{P_e} = \frac{1}{10} = 0.1\ m$

and $f_o = \frac{1}{P_o} = \frac{1}{1} = 1\ m$

Magnifying power in normal adjustment

$$m = \frac{-f_o}{f_e} = -\frac{10}{1} = -10$$

Q. (CBSE 2015) A telescope has an objective lens of focal length 15 m. If an eyepiece of focal length 1 cm is used and this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? Diameter of the moon is $3.48 \times 10^6 m$ and the radius of the Lunar orbit is $3.8 \times 10^8 m$.

Sol. Given that $f_o = 15m$, $f_e = 1\,cm = 10^{-2} m$

Angular magnification of the telescope

$$m = \frac{-f_o}{f_e} = \frac{15}{10^{-2}} = 1500$$

Angle subtended by moon at objective lens

$$\alpha = \frac{D}{\text{Radius of lunar orbit}} = \frac{D}{R}$$

Also the angle subtended by image formed by objective on itself

$$\alpha = \frac{d}{f_o} \quad [d = \text{diameter of image}]$$

$$d = \alpha \times f_o = \frac{D}{R} \times f_o$$

$$d = \frac{3.42 \times 10^6}{3.8 \times 10^8} \times 15 = 13.5 \times 10^{-2} m = 13.5\,cm$$

Q. A small telescope has an objective lens of focal length 150 cm. and an eye piece of focal length 5 cm. If this telescope is used to view a 100 m. high tower 3 Km. away, find the height of the final image when it is formed 25 cm. away from the eye piece.

CBSE 2012

Sol. Given that -

$f_o = 150$ cm. , $f_e = 5$ cm. , $D = 25$ cm.

Magnification $m = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

$$m = -\frac{150}{5}\left(1 + \frac{5}{25}\right) = -36$$

Let height of final image is h cm.

$\therefore \quad \tan\beta = \frac{h}{25}$ (β = Visual angle formed by final image at eye.

and $\tan\alpha = \frac{100}{3000} = \frac{1}{30}$ (α = Visual angle subtended by object at objective

But Magnification $m = \frac{\tan\beta}{\tan\alpha}$

$$-36 = \frac{(h/25)}{(1/30)} = \frac{30h}{25}$$

$$h = -\frac{36 \times 25}{30} = -30 \text{ cm.}$$

Here negative sign indicates the inverted image.

Q.1. The refractive index of glass with respect to water is 9/8. If the velocity and wavelength of light in glass are 2×10^8 m/s and 4000 A^0 respectively, find the velocity and wavelength of light in water.

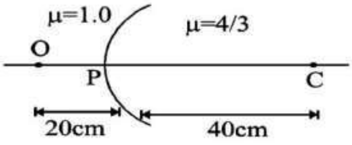
Sol: $${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{v_w}{v_g} \qquad \Rightarrow \frac{9}{8} = \frac{v_w}{2\times10^8};$$

$$v_w = \frac{9\times2\times10^8}{8} = 2.25\times10^8 \text{m/s}.$$

$${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{\lambda_w}{\lambda_g}\left(\therefore \mu_g = \frac{\lambda_0}{\lambda_g}, \mu_w = \frac{\lambda_0}{\lambda_w}\right)$$

$$\frac{9}{8} = \frac{\lambda_w}{4000}; \qquad \lambda_w = \frac{9\times4000}{8} = 4500\text{A}^0.$$

Q.2. Locate the image of the point object O. The point C is centre of curvature of the spherical surface.



Sol: Here $\mu_1 = 1, \mu_2 = 4/3, u = -20cm, R = 40cm$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{4/3}{v} - \frac{1}{-20} = \frac{4/3-1}{40}$$

$$\frac{4}{3v} + \frac{1}{20} = \frac{1}{120}$$

$$\frac{4}{3v}=\frac{1}{120}-\frac{1}{20}=\frac{-5}{120}$$

$$v=\frac{-160}{5}=-32\,cm$$

Q.3. A converging lens of focal length 5.0cm is placed in contact with a diverging lens of focal length 10.0cm. Find the combined focal length of the system.

Sol: Here $f_1=+5.0cm$ and $f_2=-10.0cm$

Therefore, the combined focal length F is given by

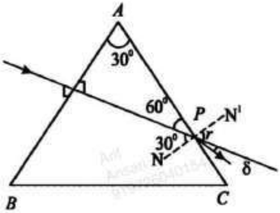
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{5.0}-\frac{1}{10.0}=+\frac{1}{10.0}$$

$\therefore\ F=+10.0cm$ ie. the combination behaves as a converging lens of focal length 10.0cm.

Q.4. A ray of light is incident normally on one of the faces of a prism of apex angles 30^0 and refractive index $\sqrt{2}$. The angle of deviation of the ray is......degree

Sol: Apply Snell's law of refraction at P

$$\frac{\sin 30^0}{\sin r}=\frac{1}{\sqrt{2}}$$



or $\sin r=\sqrt{2}\times\frac{1}{2}$

$$=\frac{1}{\sqrt{2}}=\sin 45^0$$

or $r=45^0$

$\therefore\ \delta=r-30^0=45^0-30^0=15^0$

$\therefore$ Deviation of ray$=15^0$

Q.5. If the focal length of a magnifier is 5cm calculate

a) the power of the lens

b) the magnifying power of the lens for relaxed and strained eye.

Sol: As power of a lens is reciprocal of focal length in

$$P = \frac{1}{\left(5 \times 10^{-2} m\right)} = \frac{1}{0.05} diopter = 20D$$

b) For relaxed eye, MP is minimum and will be

$$MP = \frac{D}{f} = \frac{25}{5} = 5$$

While for strained eye, MP is maximum and will be

$$MP = 1 + \frac{D}{f} = 1 + 5 = 6$$

Q.6. In a compound microscope, the object is 1cm from the objective lens. The lenses are 30 cm apart and the intermediate image is 5cm from the eye piece. What magnification is produced?

Sol: As the lenses are 30cm apart and intermediate image is formed 5cm in front of eye lens,

$u_e = 5cm$ and $v = L - u_e = 30 - 5 = 25cm$

Now as in case of compound microscope,

$$M = m_0 \times m_e = -\frac{v}{u} \times \left[\frac{D}{u_e}\right]$$

here $u = 1cm$ and $D = 25cm$

$$\text{So } M = -\frac{25}{1} \times \left[\frac{25}{5}\right] = -125$$

Negative sign implies that final image is inverted.

Q.7. A compound microscope has a magnifying power 30. The focal length of its eyepiece is 5cm. Assuming the final image to be at the least distance of distinct vision (25cm), calculate the magnification produced by objective.

Sol: In case of compound microscope,

$$M = m_0 \times m_e \rightarrow (1)$$

And in case of final image at least distance of distinct vision, $m_e = \left[1 + \frac{D}{f_e}\right] \rightarrow (2)$

so, from eqs. (1) and (2), $M = m_0\left[1 + \frac{D}{f_e}\right]$

Here $M = -30$; $D = 25cm$ and $f_e = 5cm$

So, $-30 = m_o\left[1 + \frac{25}{5}\right]$ ie $m_0 = \frac{-30}{6} = -5$

Negative sign implies that image formed by objective is inverted.

Q.8. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and eye piece is 36cm and the final image is formed at infinity. Determine the focal length of objective and eye piece.

Sol: For final image at infinity,

$M_\infty = \frac{f_0}{f_e}$ and $L_\infty = f_0 + f_e$ $\quad \therefore 5 = \frac{f_0}{f_e} \rightarrow (1)$

and $36 = f_0 + f_e \rightarrow (ii)$

Solving these two equations , we have

$f_0 = 30cm$ *and* $f_e = 6cm$